STATEMENT ON COMPETENCIES IN MATHEMATICS EXPECTED OF ENTERING COLLEGE STUDENTS

APRIL 2010

REVISED MAY 2013

Intersegmental Committee of the Academic Senates of the California Community Colleges, the California State University, and the University of California

This edition of the ICAS Statement on Competencies in Mathematics Expected of Entering College Students is dedicated to the memory of Walter Denham (1934 to 2002).
Walter represented the California Department of Education during the writing of the three previous versions. He cared deeply about mathematics education.

THE 2010 DOCUMENT HAS BEEN REVISED TO INCLUDE THE COMMON CORE STATE STANDARDS (CCSS) FOR MATHEMATICS THAT WERE ADOPTED BY THE CALIFORNIA LEGISLATURE SHORTLY AFTER PUBLICATION AND RELEASE OF THE ORIGINAL VERSION. A SECTION ON MATHEMATICAL PRACTICES HAS BEEN ADDED (SEE PART 3 ON PAGE 6), AND APPENDIX B WAS REWRITTEN TO MAP THE CCSS TO THE EXPECTATIONS OF ICAS.

ICAS Subcommittee on the Mathematics Competency Statement

Alfred Manaster (Committee Chair), UC San Diego
Joe Fiedler (Committee Co-Chair), CSU Bakersfield
Marshall Cates, CSU Los Angeles
Joan Cordova, Orange Coast College
Lipika Deka, CSU Monterey
Wade Ellis, West Valley College
Jim Greco, California Department Of Education
William Jacob, UC Santa Barbara
Abigail Leaf, Valley High School
Albert Stralka, UC Riverside
Ian Walton, Mission College

TABLE OF CONTENTS

ICAS

April 26, 2010

Dear Colleagues:

We are pleased to transmit to you the *2010 Statement on Competencies in Mathematics Expected of Entering College Students*. This document is the result of a remarkable collaboration among secondary mathematics teachers and college and university faculty. It has benefited from many comments and suggestions from people throughout California who responded to the review. It updates and replaces the previous competency statement produced in 1997. The document provides a clear statement of expectations that faculty have for the mathematical ability of students entering college in order to be successful.

The Intersegmental Committee of Academic Senates (ICAS), representing the academic senates of the three segments of California's higher education system, sponsored the efforts that produced this document. The Academic Senates of the California Community Colleges, the California State University, and the University of California all have endorsed this document and offer it as their official recommendation on math preparation to the K-12 sector, to students and their parents, to teachers and administrators, and to public policy makers.

Please share this statement with your colleagues, distribute it widely, or refer interested parties to the ICAS website to download the document: http://icas-ca.org/.

Sincerely,

Jane Patton, President	John Tarjan, Chair	Henry Powell, Chair
CCC Academic Senate	CSU Academic Senate	UC Academic Senate

INTRODUCTION

The goal of this Statement on Competencies in Mathematics Expected of Entering College Students is to provide a clear and coherent message about the mathematics that students need to know and to be able to do to be successful in college. While parts of this Statement were written with certain audiences in mind, the document as a whole should be useful for anyone who is concerned about the preparation of California's students for college. This represents an effort to be realistic about the skills, approaches, experiences, and subject matter that make up an appropriate mathematical background for entering college students.

"Entering College Students" in general refers to students who enter a California postsecondary institution with the goal of receiving a bachelor's degree. However, it is important that students who plan to enter a California community college be aware that a wide variety of courses exist to help them transition from lower mathematical skill levels to the competencies described in this document. Most community colleges offer a wide range of mathematics courses including some as elementary as arithmetic.

The first section describes some characteristics that identify the student who is properly prepared for college courses that are quantitative in their approach. The second section describes the subject matter that is an essential part of the background for all entering college students, as well as describing what is the essential background for students intending quantitative majors. Among the descriptions of subject matter there are sample problems. These are intended to clarify the descriptions of subject matter and to be representative of the appropriate level of understanding. The sample problems do not cover all of the mathematical topics identified.

No section of this Statement should be ignored. Students need the approaches, attitudes, and perspectives on mathematics described in the first section. Students also need extensive skills and knowledge in the subject matter areas described in the second section. Inadequate attention to either of these components is apt to disadvantage the student in ways that impose a serious impediment to success in college. Nothing less than a balance among these components is acceptable for California's students.

The discussion in this document of the mathematical competencies expected of entering college students is predicated on the following basic recommendation:

> *For proper preparation for baccalaureate level course work, all students should be enrolled in a mathematics course in every semester of high school. It is particularly important that students take mathematics courses in their senior year of high school, even if they have completed three years of college preparatory mathematics by the end of their junior year. Experience has shown that students who take a hiatus from the study of mathematics in high school are very often unprepared for courses of a quantitative nature in college and are unable to continue in these courses without remediation in mathematics.*

SECTION 1

Approaches to Mathematics

This section enumerates characteristics of entering freshmen college students who have the mathematical maturity to be successful in their first college mathematics course, and in other college courses that are quantitative in their approach. A student's first college mathematics course will depend upon the student's goals and preparation. These characteristics are described primarily in terms of how students approach mathematical problems. The second part of this section provides suggestions to secondary teachers of ways to present mathematics that will help their students to develop these characteristics.

Part 1: Dispositions of well-prepared students toward mathematics

Crucial to their success in college is the way in which students encounter the challenges of new problems and new ideas. From their high school mathematics courses students should have gained certain approaches, attitudes, and perspectives:

- A view that mathematics makes sense—students should perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized and applied.
- An ease in using their mathematical knowledge to solve unfamiliar problems in both concrete and abstract situations—students should be able to find patterns, make conjectures, and test those conjectures; they should recognize that abstraction and generalization are important sources of the power of mathematics; they should understand that mathematical structures are useful as representations of phenomena in the physical world; they should consistently verify that their solutions to problems are reasonable.
- A willingness to work on mathematical problems requiring time and thought, problems that are not solved by merely mimicking examples that have already been seen—students should have enough genuine success in solving such problems to be confident, and thus to be tenacious, in their approach to new ones.
- A readiness to discuss the mathematical ideas involved in a problem with other students and to write clearly and coherently about mathematical topics—students should be able to communicate their understanding of mathematics with peers and teachers using both formal and natural languages correctly and effectively.
- An acceptance of responsibility for their own learning—students should realize that their minds are their most important mathematical resource, and that teachers and other students can help them to learn but can't learn for them.
- The understanding that assertions require justification based on persuasive arguments, and an ability to supply appropriate justifications—students should habitually ask "Why?" and should have a familiarity with reasoning at a variety of levels of formality, ranging from concrete examples

through informal arguments using words and pictures to precise structured presentations of convincing arguments.

- While proficiency in the use of technology is not a substitute for mathematical competency, students should be familiar with and confident in the use of computational devices and software to manage and display data, to explore functions, and to formulate and investigate mathematical conjectures.
- A perception of mathematics as a unified field of study—students should see interconnections among various areas of mathematics, which are often perceived as distinct.

Part 2: Aspects of Mathematics Instruction to Foster Student Understanding and Success

There is no best approach to teaching, not even an approach that is effective for all students, or for all instructors. One criterion that should be used in evaluating approaches to teaching mathematics is the extent to which they lead to the development in the student of the dispositions, concepts, and skills that are crucial to success in college. Various technologies can be used to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When strategically used, technology can improve student access to mathematics. It should be remembered that in the mathematics classroom, time spent focused on mathematics is crucial. The activities and behaviors that can accompany the learning of mathematics must not become goals in themselves—understanding of mathematics is always the goal.

While much has been written recently about approaches to teaching mathematics, as it relates to the preparation of students for success in college, there are a few aspects of mathematics instruction that merit emphasis here.

Modeling Mathematical Thinking

Students are more likely to become intellectually venturesome if it is not only expected of them, but if their classroom is one in which they see others, especially their teacher, thinking in their presence. It is valuable for students to learn with a teacher and others who get excited about mathematics, who work as a team, who experiment and form conjectures. They should learn by example that it is appropriate behavior for people engaged in mathematical exploration to follow uncertain leads, not always to be sure of the path to a solution, and to take risks. Students should understand that learning mathematics is fundamentally about inquiry and personal involvement.

Solving Problems

Problem solving is the essence of mathematics. Problem solving is not a collection of specific techniques to be learned; it cannot be reduced to a set of procedures. Problem solving is taught by giving students appropriate experience in solving unfamiliar problems, by then engaging them in a discussion of their various attempts at solutions, and by reflecting on these processes. Students entering college should have had successful experiences solving a wide variety of mathematical problems. The goal is the development of open, inquiring, and demanding minds. Experience in solving problems gives students the confidence and skills to approach new situations creatively, by modifying, adapting, and combining their mathematical tools; it gives students the determination to refuse to accept an answer until they can explain it.

Developing Analytic Ability and Logic

A student who can analyze and reason well is a more independent and resilient student. The instructional emphasis at all levels should be on a thorough understanding of the subject matter and the development of logical reasoning. Students should be asked "Why?" frequently enough that they anticipate the question, and ask it of themselves. They should be expected to construct compelling arguments to explain why, and to understand a proof comprising a significant sequence of implications. They should be expected to question and to explore why one statement follows from another. Their understandings should be challenged with questions that cause them to further examine their reasoning. Their experience with mathematical proof should not be limited to the format of a two-column proof; rather, they should see, understand, and construct proofs in various formats throughout their course work. A classroom full of discourse and interaction that focuses on reasoning is a classroom in which analytic ability and logic are being developed.

Experiencing Mathematics in Depth

Students who have seen a lot but can do little are likely to experience difficulty in college. While there is much that is valuable to know in the breadth of mathematics, a shallow but broad mathematical experience does not develop the sort of mathematical sophistication that is most valuable to students in college. Emphasis on coverage of too many topics can trivialize the mathematics that awaits the students, turn the study of mathematics into the memorization of discrete facts and skills, and divest students of their curiosity. By delving deeply into well-chosen areas of mathematics, students develop not just the self-confidence but the ability to understand other mathematics more readily, and independently.

Appreciating the Beauty and Fascination of Mathematics

Students who spend years studying mathematics yet never develop an appreciation of its beauty are cheated of an opportunity to become fascinated by ideas that have engaged individuals and cultures for thousands of years. While applications of mathematics are valuable for motivating students, and as paradigms for their mathematics, an appreciation for the inherent beauty of mathematics should also be nurtured, as mathematics is valuable for more than its utility. Opportunities to enjoy mathematics can make the student more eager to search for patterns, for connections, for answers. This can lead to a deeper mathematical understanding, which also enables the student to use mathematics in a greater variety of applications. An appreciation for the aesthetics of mathematics should permeate the curriculum and should motivate the selection of some topics.

Building Confidence

For each student, successful mathematical experiences are self-perpetuating. It is critical that student confidence be built upon genuine successes—false praise usually has the opposite effect. Genuine success can be built in mathematical inquiry and exploration. Students should find support and reward for being inquisitive, for experimenting, for taking risks, and for being persistent in finding solutions they fully understand. An environment in which this happens is more likely to be an environment in which students generate confidence in their mathematical ability.

Communicating

While solutions to problems are important, so are the processes that lead to the solutions and the reasoning behind the solutions. Students should be able to communicate all of this, but this ability is not quickly developed. Students need extensive experiences in oral and written communication regarding mathematics, and they need constructive, detailed feedback in order to develop these skills. Mathematics is, among other things, a language, and students should be comfortable using the language of mathematics. The goal is not for students to memorize an extensive mathematical vocabulary, but rather for students to develop ease in carefully and precisely discussing the mathematics they are learning. Memorizing terms that students don't use does not contribute to their mathematical understanding. However, using appropriate terminology so as to be precise in communicating mathematical meaning is part and parcel of mathematical reasoning.

Becoming Fluent in Mathematics

To be mathematically capable, students must have a facility with the basic techniques of mathematics. There are necessary skills and knowledge that students must routinely exercise without hesitation. Mathematics is the language of the sciences, and thus fluency in this language is a basic skill. College mathematics classes require that students bring with them ease with the standard skills of mathematics that allows them to focus on the ideas and not become lost in the details. However, this level of internalization of mathematical skills should not be mistaken for the only objective of secondary mathematics education. Student understanding of mathematics is the goal. In developing a skill, students first must develop an understanding. Then as they use the skill in different contexts, they gradually wean themselves from thinking about it deeply each time, until its application becomes routine. But their understanding of the mathematics is the map they use whenever they become disoriented in this process. The process of applying skills in varying and increasingly complex applications is one of the ways that students not only sharpen their skills, but also reinforce and strengthen their understanding. Thus, in the best of mathematical environments, there is no dichotomy between gaining skills and gaining understanding. A curriculum that is based on depth and problem solving can be quite effective in this regard provided that it focuses on appropriate areas of mathematics.

Part 3: The Common Core State Standards of Mathematical Practice

The Common Core State Standards were adopted by the California State Board of Education in 2010 and were updated in 2013. They are available online at http://www.cde.ca.gov/re/cc/. Like this document, in addition to identifying key content, the CCSS contain eight *Standards of Mathematical Practice* that "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students." These Standards encompass most of the Aspects of Mathematics Instruction and should enable development of the Dispositions of Well-Prepared Students Toward Mathematics discussed here. These eight standards are:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

The authors of the CCSS cite as background for their Standards of Practice the NCTM Process Standards: problem solving, reasoning and proof, communication, representation, and connections; and they also cite the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). Both the NCTM Standards and *Adding It Up*, along with the CCSS, are grounded in research and their development included substantial consultation with K-12 teachers and college mathematics faculty. As such, all are worthy of study and collectively they describe a consensus about the levels of reasoning and engagement necessary for college readiness in mathematics and quantitative literacy necessary for all college-level work. Taken together the Standards of Mathematical Practice should be viewed as an integrated whole where each component should be visible in every unit of instruction.

When preparing mathematics course descriptions for UC/CSU area 'c' approval (the Mathematics section of the 'a-g' requirements) the Standards of Mathematical Practice must be discussed in the application template *Key Assignments, Instructional Methods and/or Strategies*, and *Assessment* sections. The Approaches to Mathematics discussed in this section provide a way to think about preparing the application. For example, do the assignments expect students to work on problems requiring time and thought that are not solved by merely mimicking examples that have already been seen? Does instruction model mathematical thinking where justification is based upon persuasive arguments? Do the assessments require that students communicate their reasoning? Courses redesigned for Common Core alignment will have to be resubmitted for area 'c' approval, and discussion of how the Standards of Mathematical Practice are implemented is an essential ingredient of a successful application.

SECTION 2

Subject Matter

Decisions about the subject matter for secondary mathematics courses are often difficult, and are too-easily based on tradition and partial information about the expectations of the colleges. What follows is a description of mathematical areas of focus that are (1) essential for all entering college students; (2) desirable for all entering college students; (3) essential for college students to be adequately prepared for quantitative majors; and (4) desirable for college students who intend quantitative majors. This description of content will in many cases necessitate adjustments in a high school mathematics curriculum, generally in the direction of deeper study in the more important areas, at the expense of some breadth of coverage.

Sample problems have been included to indicate the appropriate level of understanding for some areas. The problems included do not cover all of the mathematical topics described, and many involve topics from several areas. Entering college students working independently should be able to solve problems like these in a short time—less than half an hour for each problem included. Students must also be able to solve more complex problems requiring significantly more time.

Part 1: Essential areas of focus for all entering college students

What follows is a summary of the mathematical subjects that are an essential part of the knowledge base and skill base for all students who enter higher education. Students are best served by deep mathematical experiences in these areas. This is intended as a brief compilation of the truly essential topics, as opposed to topics to which students should have been introduced but need not have mastered. The skills and content knowledge that are prerequisite to high school mathematics courses are of course still necessary for success in college, although they are not explicitly mentioned here. Students who lack these skills on leaving high school may acquire them through some community college courses.

- Variables, Equations, and Algebraic Expressions: Algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols; solutions of linear equations and inequalities; absolute value; powers and roots; solutions of quadratic equations; solving two linear equations in two unknowns, including the graphical interpretation of a simultaneous solution. Emphasis should be placed on algebra both as a language for describing mathematical relationships and as a means for solving problems; algebra should not merely be the implementation of a set of rules for manipulating symbols.

> The braking distance of a car (how far it travels after the brakes are applied until it comes to a stop) is proportional to the square of its speed.
>
> Write a formula expressing this relationship and explain the meaning of each term in the formula.
>
> If a car traveling 50 miles per hour has a braking distance of 105 feet, then what would its braking distance be if it were traveling 60 miles per hour?

> Solve for x and give a reason for each step: $\frac{2}{3x+1} + 2 = \frac{2}{3}$

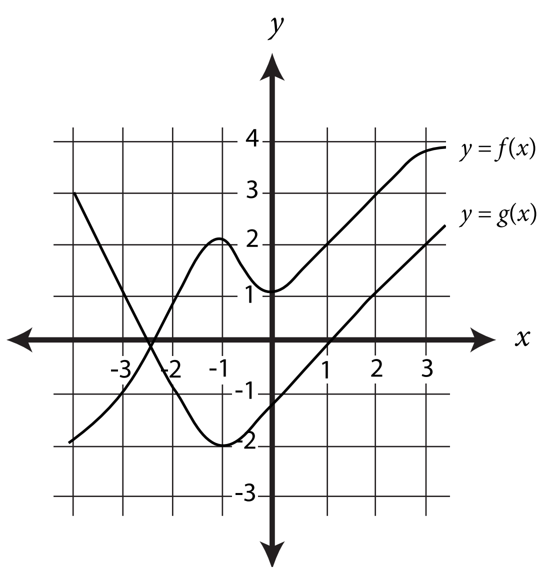
United States citizens living in Switzerland must pay taxes on their income to both the United States and to Switzerland. The United States tax is 28% of their taxable income after deducting the tax paid to Switzerland. The tax paid to Switzerland is 42% of their taxable income after deducting the tax paid to the United States. If a United States citizen living in Switzerland has a taxable income of $75,000, how much tax must that citizen pay to each of the two countries? Find these values in as many different ways as you can; try to find ways both using and not using graphing calculators. Explain the methods you use.

- *Families of Functions and Their Graphs:* Applications; linear functions; quadratic and power functions; exponential functions; roots; operations on functions and the corresponding effects on their graphs; interpretation of graphs; function notation; functions in context, as models for data. Emphasis should be placed on various representations of functions—using graphs, tables, variables, and words—and on the interplay among the graphical and other representations; repeated manipulations of algebraic expressions should be minimized.

Car dealers use the "rule of thumb" that a car loses about 30% of its value each year.

Suppose that you bought a new car in December 1995 for $20,000. According to this "rule of thumb," what would the car be worth in December 1996? In December 1997? In December 2005?

Develop a general formula for the value of the car t years after purchase.



(a) Which is larger, $f(-3)$ or $f(3)$

(b) Which among the following three quantities is the largest?

$f(-1) - g(-1)$?

$f(0) - g(0)$?

$f(1) - g(1)$?

(c) For which values of x does $g(x) = f(-3)$?

(d) Find a value of x for which $f(x) = g(x+2)$

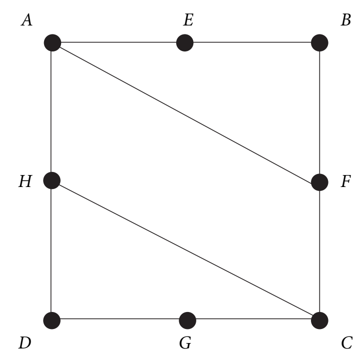
Find a quadratic function of x that has zeroes at $x = -1$ and $x = 2$.

Find a cubic function of x that has zeroes at $x = -1$ and $x = 2$ and nowhere else.

- *Geometric Concepts:* Distances, areas, and volumes, and their relationship with dimension; angle measurement; similarity; congruence; lines, triangles, circles, and their properties; symmetry; Pythagorean Theorem; coordinate geometry in the plane, including distance between points, midpoint, equation of a circle; introduction to coordinate geometry in three dimensions. Emphasis should be placed on developing an understanding of geometric concepts sufficient to solve unfamiliar problems and an understanding of the need for compelling geometric arguments; mere memorization of terminology and formulas should receive as little attention as possible.

A contemporary philosopher wrote that in 50 days the earth traveled approximately 40 million miles along its orbit and that the distance between the positions of the earth at the beginning and the end of the 50 days was approximately 40 million miles. Discuss any errors you can find in these conclusions or explain why they seem to be correct. You may approximate the earth's orbit by a circle with radius 93 million miles.

ABCD is a square and the midpoints of the sides are *E, F, G,* and *H*. *AB* = 10 in. Use at least two different methods to find the area of parallelogram AFCH.



Two trees are similar in shape, but one is three times as tall as the other.

If the smaller tree weighs two tons, how much would you expect the larger tree to weigh?

Suppose that the bark from these trees is broken up and placed into bags for landscaping uses. If the bark from these trees is the same thickness on the smaller tree as the larger tree, and if the larger tree yields 540 bags of bark, how many bags would you expect to get from the smaller tree?

An 82 in. by 11 in. sheet of paper can be rolled lengthwise to make a cylinder, or it can be rolled widthwise to make a different cylinder.

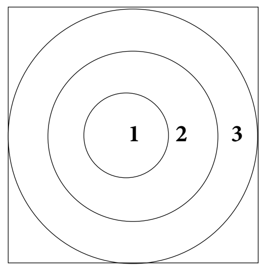
Without computing the volumes of the two cylinders, predict which will have the greater volume, and explain why you expect that.

Find the volumes of the two cylinders to see if your prediction was correct.

If the cylinders are to be covered top and bottom with additional paper, which way of rolling the cylinder will give the greater total surface area?

- *Probability:* Counting (permutations and combinations, multiplication principle); sample spaces; expected value; conditional probability; independence; area representations of probability. Emphasis should be placed on a conceptual understanding of discrete probability; aspects of probability that involve student memorization and rote application of formulas should be minimized.

> If you take one jelly bean from a large bin containing 10 lbs. of jelly beans, the chance that it is cherry flavored is 20%. How many more pounds of cherry jelly beans would have to be mixed into the bin to make the chance of getting a cherry jelly bean 25%?

> A point is randomly illuminated on a computer game screen that looks like the figure shown below.
>
> 
>
>
> The radius of the inner circle is 3 inches; the radius of the middle circle is 6 inches; the radius of the outer circle is 9 inches.
>
> What is the probability that the illuminated point is in region 1?
>
> What is the probability that the illuminated point is in region I if you know that it isn't in region 2?

> A fundraising group sells 1000 raffle tickets at $5 each. The first prize is an $1,800 computer. Second prize is a $500 camera and the third prize is $300 cash. What is the expected value of a raffle ticket?

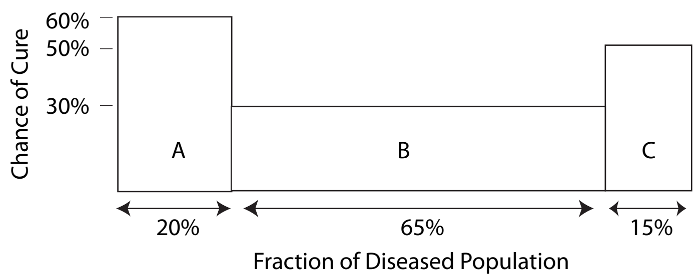
> Ashley, Frank, Jose, Mercedes, and Wade will line up in random order at a movie theater. What is the probability that Ashley and Mercedes stand next to each other?

- *Data Analysis and Statistics:* Presentation and analysis of data; measures of center such as mean and median, and measures of spread such as standard deviation and interquartile range; representative samples; using lines to fit data and make predictions. Emphasis should be placed on organizing and describing data, interpreting summaries of data, and making predictions based on the data, with common sense as a guide; algorithms should be learned with an understanding of the underlying ideas.

The table at the right shows the population of the USA in each of the last five censuses. Make a scatter plot of this data and draw a line on your scatter plot that fits this data well. Find an equation for your line, and use this equation to predict what the population was in the year 1975. Plot that predicted point on your graph and see if it seems reasonable. What is the slope of your line? Write a sentence that describes to someone who might not know about graphs and lines what the meaning of the slope is in terms involving the USA population.

Year	USA Population (Millions)
1960	180.7
1970	205.1
1980	227.7
1990	249.9
2000	281.4

The results of a study of the effectiveness of a certain treatment for a blood disease are summarized in the chart shown below. The blood disease has three types, A, B, and C. The cure rate for each of the types is shown vertically on the chart. The percentage of diseased persons with each type of the disease is shown horizontally on the same chart.



For which type of the disease is the treatment most effective?

From which type of the disease would the largest number of patients be cured by the treatment?

What is the average cure rate of this treatment for all of the persons with the disease?

Jane was on her computer every day one week for the number of hours listed. Find the mean and standard deviation of the time she was on her computer that week.

12, 4, 5, 6, 8, 5, 9

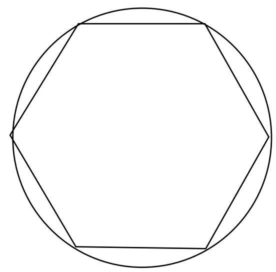
Make up another list of seven numbers with the same mean and a smaller standard deviation.

Make up another list of seven numbers with the same mean and a larger standard deviation.

- *Argumentation and Proof:* Logical implication; hypotheses and conclusions; inductive and deductive reasoning. Emphasis should be placed on constructing and recognizing valid mathematical arguments; mathematical proofs should not be considered primarily as formal exercises.

Select any odd number, then square it, and then subtract one. Must the result always be even? Write a convincing argument.

Use the perimeter of a regular hexagon inscribed in a circle to explain why $\pi > 3$.



Does the origin lie inside of, outside of, or on the geometric figure whose equation is $x^2 + y^2 - 10x + 10y - 1 = 0$? Explain your reasoning.

Part 2: Desirable areas of focus for all entering college students

What follows is a brief summary of some of the mathematical subjects that are a **desirable** part of the mathematical experiences for all students who enter higher education. No curriculum would include study in all of these areas, as that would certainly be at the expense of opportunities for deep explorations in selected areas. But these areas provide excellent contexts for the approaches to teaching suggested in Section I, and any successful high school mathematics program will include some of these topics. The emphasis here is on enrichment and on opportunities for student inquiry.

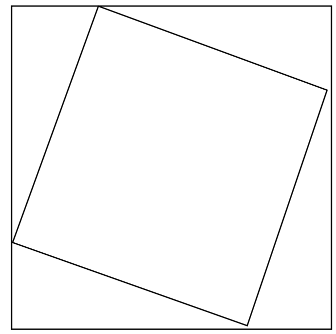
- *Discrete Mathematics:* Topics such as set theory, graph theory, coding theory, voting systems, game theory, and decision theory.
- *Sequences and Series:* Geometric and arithmetic sequences and series; the Fibonacci sequence; recursion relations.
- *Geometry:* Right triangle trigonometry; transformational geometry including dilations; tessellations; solid geometry; three-dimensional coordinate geometry, including lines and planes.
- *Number Theory:* Prime numbers; prime factorization; rational and irrational numbers; triangular numbers; Pascal's triangle; Pythagorean triples.

Part 3: Essential areas of focus for students in quantitative majors

What follows is a brief summary of the mathematical subjects that are an **essential** part of the knowledge base and skill base for students to be adequately prepared for science, technology, engineering, and mathematics (STEM) majors. At the very least, any entering college student considering a STEM major should be well prepared to begin a calculus sequence for physical sciences and engineering majors. Students are best served by deep experiences in these mathematical subjects. The skills and content knowledge listed above as essential for all students entering college are of course also essential for these students—moreover, students in quantitative majors must have a deeper understanding of and a greater facility with those areas.

- *Variables, Equations, and Algebraic Expressions:* Solutions to systems of equations, and their geometrical interpretation; solutions to quadratic equations, both algebraic and graphical; complex numbers and their arithmetic; the correspondence between roots and factors of polynomials; rational expressions; the binomial theorem.

In the figure shown to the right, the area between the two squares is 11 square inches. The sum of the perimeters of the two squares is 44 inches. Find the length of a side of the larger square.



Determine the middle term in the binomial expansion of $\left(x - \frac{2}{x}\right)^{10}$

- *Functions:* Rational functions; logarithmic functions, their graphs, and applications; trigonometric functions of real variables, their graphs, properties including periodicity, and applications to right triangle trigonometry; basic trigonometric identities; operations on functions, including addition, subtraction, multiplication, reciprocals, division, composition, and iteration; inverse functions and their graphs; domain and range.

Which of the following functions are their own inverses? Use at least two different methods to answer this, and explain your methods.

$f(x) = \frac{2}{x}$ $\qquad g(x) = x^3 + 4$

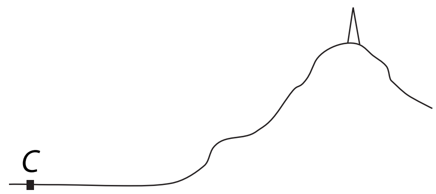
$h(x) = \frac{2 + \ln(x)}{2 - \ln(x)}$ $\qquad k(x) = \sqrt[3]{\frac{x^3 + 1}{x^3 - 1}}$

Scientists have observed that living matter contains, in addition to Carbon, C-12, a fixed percentage of a radioactive isotope of Carbon, C-14. When the living material dies, the amount of C-12 present remains constant, but the amount of C-14 decreases exponentially with a half life of 5,550 years. In 1965, the charcoal from cooking pits found at a site in Newfoundland used by Vikings was analyzed and the percentage of C-14 remaining was found to be 88.6%. What was the approximate date of this Viking settlement?

Find all quadratic functions of x that have zeroes at $x = -1$ and $x = 2$.

Find all cubic functions of x that have zeroes at $x = -1$ and $x = 2$ and nowhere else.

A cellular phone system relay tower is located atop a hill. You can measure angles and have a calculator. You are standing at point *C*. Assume that you have a clear view of the base of the tower from point *C*, that *C* is at sea level, and that the top of the hill is 2000 ft. above sea level.



Describe a method that you could use for determining the height of the relay tower, without going to the top of the hill. Next choose some values for the unknown measurements that you need in order to find a numerical value for the height of the tower, and find the height of the tower.

- *Geometric Concepts:* Two- and three-dimensional coordinate geometry; locus problems; polar coordinates; vectors; parametric representations of curves.

Find any points of intersection (first in polar coordinates and then in rectangular coordinates) of the graph of $r = 1 + \sin\theta$ and the circle of radius centered at the origin. Verify your solutions by graphing the curves.

Find any points of intersection (first in polar coordinates and then in rectangular coordinates) of the graph of $r = 1 + \sin\theta$ and the line with slope 1 that passes through the origin. Verify your solutions by graphing the curves.

Marcus is in his back yard, and has left his stereo and a telephone 24 feet apart. He can't move the stereo or the phone, but he knows from experience that in order to hear the telephone ring, he must be located so that the stereo is at least twice as far from him as the phone. Draw a diagram with a coordinate system chosen, and use this to find out where Marcus can be in order to hear the phone when it rings.

A box is twice as high as it is wide and three times as long as it is wide. It just fits into a sphere of radius 3 feet. What is the width of the box?

- *Argumentation and Proof:* Mathematical implication; mathematical induction and formal proof. Attention should be paid to the distinction between plausible or informal reasoning and complete or rigorous demonstrations.

Select any odd number, then square it, and then subtract one. Must the result always be divisible by 4? Must the result always be divisible by 8? Must the result always be divisible by 16? Write convincing arguments or give counterexamples.

The midpoints of a quadrilateral are connected to form a new quadrilateral. Prove that the new quadrilateral must be a parallelogram.

In case the first quadrilateral is a rectangle, what special kind of parallelogram must the new quadrilateral be? Explain why your answer is correct for any rectangle.

Part 4: Desirable areas of focus for students in quantitative majors

What follows is a brief summary of some of the mathematical subjects that are a **desirable** part of the mathematical experiences for students who enter higher education with the possibility of pursuing STEM majors. No curriculum would include study in all of these areas, as that would certainly be at the expense of opportunities for deep explorations in selected areas. But these areas each provide excellent contexts for the approaches to teaching suggested in Section 1. The emphasis here is on enrichment and on opportunities for student inquiry.

- *Vectors and Matrices:* Vectors in the plane; vectors in space; dot product and cross product; matrix operations and applications.
- *Probability and Statistics:* Distributions as models; discrete distributions, such as the Binomial Distribution; continuous distributions, such as the Normal Distribution; fitting data with curves; correlation, regression; sampling, graphical displays of data.
- *Conic Sections:* Representations as plane sections of a cone; focus-directrix properties; reflective properties.
- *Non-Euclidean Geometry:* History of the attempts to prove Euclid's parallel postulate; equivalent forms of the parallel postulate; models in a circle or sphere; seven-point geometry.
- *Calculus:* A high school calculus course should have the same depth, rigor and content as university calculus courses designed for physical sciences and engineering majors. Prior to taking the course, students should have successfully completed four years of secondary school mathematics. Students completing the course should take one of the College Board's Advanced Placement Calculus examinations.

COMMENTS ON IMPLEMENTATION

Students who are ready to succeed in college will have become prepared throughout their primary and secondary education, not just in their college preparatory high school classes. Concept and skill development in the high school curriculum should be a deliberately coordinated extension of the elementary and middle school curriculum. This will require some changes, and some flexibility, in the planning and delivery of curriculum, especially in the first three years of college preparatory mathematics. For example, student understanding of probability and data analysis will be based on experiences that began when they began school, where they became accustomed to performing experiments, collecting data, and presenting the data. This is a more substantial and more intuitive understanding of probability and data analysis than one based solely on an axiomatic development of probability functions on a sample space, for example. It must be noted that inclusion of more study of data analysis in the first three years of the college preparatory curriculum, although an extension of the K-8 curriculum, will be at the expense of some other topics. The general direction, away from a broad but shallow coverage of algebra and geometry topics, should allow opportunities for this.

APPENDIX A

Some Mathematical Skills Necessary for College Work

What follows is a collection of skills that students must routinely exercise without hesitation in order to be prepared for college work. These are intended as indicators—students who have difficulty with many of these skills are significantly disadvantaged and are apt to require remediation in order to succeed in college courses. This is not an exhaustive list of basic skills. It is also not a list of skills that are sufficient to ensure success in college mathematical endeavors.

The absence of errors in student work is not the litmus test for mathematical preparation. Many capable students will make occasional errors in performing the skills listed below, but they should be in the habit of checking their work and thus readily recognize these mistakes, and should easily access their understanding of the mathematics in order to correct them.

1. Perform arithmetic with signed numbers, including fractions and percentages.
2. Combine like terms in algebraic expressions.
3. Use the distributive law for monomials and binomials.
4. Factor monomials out of algebraic expressions.
5. Solve linear equations of one variable.
6. Solve quadratic equations of one variable.
7. Apply laws of exponents.
8. Plot points that are on the graph of a function.
9. Given the measures of two angles in a triangle, find the measure of the third.
10. Find areas of right triangles.
11. Find and use ratios from similar triangles.
12. Given the lengths of two sides of a right triangle, find the length of the third side.

APPENDIX B

Summaries of Subject Matter Topics with Related California and NCTM Standards

This appendix lists the summaries of the subject matter topics presented in Section 2 of the Statement. After each summary, citations of related California Standards (from the *California Common Core State Standards for Mathematics*, adopted by the California State Board of Education August 2010[1]) and the NCTM standards (from *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, 2000[2]) are given. There are two reasons for including these citations. One is to show the relationship between the Expected Competencies and the state and national standards. The second is to help teachers and other readers of the Expected Competencies find fuller descriptions of them.

The citations of the California Common Core State Standards include abbreviations of grade level or high school strand followed by the relevant subsection numbers.

The citations of the NCTM standards are grade-band specific expectations of content standards as they appear in the Appendix on pages 392-401 of Principles and Standards. In order to save space in this document, the standards are specified by their content area and a brief description consisting of some of their keywords.

Part 1

Essential areas of focus for all entering college students.

Variables, Equations, and Algebraic Expressions

- Algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols; solutions of linear equations and inequalities; absolute value; powers and roots; solutions of quadratic equations; solving two linear equations in two unknowns including the graphical interpretation of a simultaneous solution. Emphasis should be placed on algebra both as a language for describing mathematical relationships and as a means for solving problems; algebra should not merely be the implementation of a set of rules for manipulating symbols.

CA Common Core State Standards

7.EE: Use properties of operations to generate equivalent expressions. #1, #2

7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. #3, #4

---

[1] *Common Core State Standards for Mathematics for California Public Schools Kindergarten Through Grade Twelve,* copyright 2010 by the California Department of Education. Available at *http://www.cde.ca.gov/re/cc*

[2] Reprinted with permission from *Principles and Standards for School Mathematics,* copyright 2000 by the National Council of Teachers of Mathematics. All rights reserved. NCTM does not endorse the content or validity of these alignments.

8.EE: Work with radicals and integer exponents. #1, #2, #3, #4

8.EE: Understand the connections between proportional relationships, lines, and linear equations. #5, #6

8.EE: Analyze and solve linear equations and pairs of simultaneous linear equations. #7, #8

A-SSE: Interpret the structure of expressions. #1, #2

A-SSE: Write expressions in equivalent forms to solve problems. #3

A-APR: Perform arithmetic operations on polynomials. #1

A-APR: Understand the relationship between zeros and factors of polynomials. #2

A-CED: Create equations that describe numbers or relationships. #1

A-REI: Understand solving equations as a process of reasoning and explain the reasoning. #1, #2

A-REI: Solve equations and inequalities in one variable. #3, #4

A-REI: Solve systems of equations. #6

A-REI: Represent and solve equations and inequalities graphically. #10, #12

NCTM Standards

AL: Patterns: 9-12: generalize patterns using explicitly defined and recursively defined functions

AL: Patterns: 6-8: represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

AL: Symbols: 9-12: Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations

AL: Symbols: 9-12: Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases

AL: Symbols: 9-12: Use symbolic algebra to represent and explain mathematical relationships

AL: Symbols: 9-12: judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

AL: Symbols: 6-8: recognize and generate equivalent forms for simple algebraic expressions and solve linear equations

Families of Functions and Their Graphs

- Applications; linear functions; quadratic and power functions; exponential functions; roots; operations on functions and the corresponding effects on their graphs; interpretation of graphs; function notation; functions in context, as models for data. Emphasis should be placed on various representations of functions—using graphs, tables, variables, and words—and on the interplay among the graphical and other representations; repeated manipulations of algebraic expressions should be minimized.

CA Common Core State Standards

8.F Define, evaluate, and compare functions. #1, #2, #3

8.F Use functions to model relationships between quantities. #4, #5

F-IF: Understand the concept of a function and use function notation. #1, #2

F-IF: Interpret functions that arise in applications in terms of the context. #4, #5

F-IF: Analyze functions using different representations. #7a,b #8a,b #9

F-BF: Build a function that models a relationship between two quantities. #1a

F-BF: Build new functions from existing functions. #3

F-LE: Construct and compare linear, quadratic, and exponential models and solve problems. #1, #2, #3

F-LE: Interpret expressions for functions in terms of the situation they model. #5

NCTM Standards

NO: Understand operations: 9-12: judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities

AL: Patterns: 9-12: understand relations and functions and select, convert flexibly among, and use various representations for them

AL: Patterns: 9-12: analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior

AL: Patterns: 9-12: understand and compare the properties of classes of functions, including exponential, polynomial, functions

AL: Patterns: 6-8: identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations

AL: Relationships: 9-12: identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Geometric Concepts

- Distances, areas, and volumes, and their relationship with dimension; angle measurement; similarity; congruence; lines, triangles, circles, and their properties; symmetry; Pythagorean Theorem; coordinate geometry in the plane, including distance between points, midpoint, equation of a circle; introduction to coordinate geometry in three dimensions. Emphasis should be placed on developing an understanding of geometric concepts sufficient to solve unfamiliar problems and an understanding of the need for compelling geometric arguments; mere memorization of terminology and formulas should receive as little attention as possible.

CA Common Core State Standards

8.G Understand congruence and similarity using physical models, transparencies, or geometry software. #1, #2, #3, #4, #5

8.G Understand and apply the Pythagorean Theorem. #6, #7, #8

8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. #9

G-CO: Experiment with transformations in the plane. #1, #2, #3, #4, #5

G-CO: Understand congruence in terms of rigid motions. #6, #7, #8

G-CO: Prove geometric theorems. #9, #10, #11

G-CO: Make geometric constructions. #12, #13

G-SRT: Apply trigonometry to general triangles. #9, #10, #11

G-C: Understand and apply theorems about circles. #1, #2, #3, #4

G-C: Find arc lengths and areas of sectors of circles. #5

G-GPE: Use coordinates to prove simple geometric theorems algebraically. #4, #5, #6

G-GMD Explain Volume formulas and use them to solve problems. #3

NCTM Standards

GM: Synthetic: 9-12: Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them

GM: Synthetic: 6-8: Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects

GM: Analytic: 9-12: investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates

GM: Transformations: 6-8: examine the congruence, similarity, and line or rotational symmetry of objects using transformations

MS: Systems: 6-8: understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume

MS: Tools: 9-12: understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Probability

- Counting (permutations and combinations, multiplication principle); sample spaces; expected value; conditional probability; independence; area representations of probability. Emphasis should be placed on a conceptual understanding of discrete probability; aspects of probability that involve memorization and rote application of formulas should be minimized.

CA Common Core State Standards

7-SP Investigate chance processes and develop, use, and evaluate probability models. #5, #6, #7, #8

S-CP: Understand independence and conditional probability and use them to interpret data. #1, #2, #3, #4, #5

S-CP: Use the rules of probability to compute probabilities of compound events in a uniform probability model. #6, #7

S-MD Calculate expected values and use them to solve problems. #1, #2

NCTM Standards

NO: Understand operations: 9-12: develop an understanding of permutations and combinations as counting techniques

DA: Probability: 9-12: understand the concepts of sample space and construct sample spaces in simple cases

DA: Probability: 9-12: compute and interpret the expected value of random variables in simple cases

DA: Probability: 9-12: understand the concepts of conditional probability and independent events

DA: Probability: 6-8: compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models

Data Analysis and Statistics

- *Data Analysis and Statistics:* Presentation and analysis of data; measures of center such as mean and median, and measures of spread such as standard deviation and interquartile range; representative samples; using lines to fit data and make predictions. Emphasis should be placed on organizing and describing data, interpreting summaries of data, and making predictions based on the data, with common sense as a guide; algorithms should be learned with an understanding of the underlying ideas.

CA Common Core State Standards

8.SP Investigate patterns of association in bivariate data. #1, #2, #3, #4

S-ID: Summarize, represent, and interpret data on a single count or measurement variable. #1, #2, #3

S-ID: Summarize, represent, and interpret data on two categorical and quantitative variables. #6

S-ID: Interpret linear models. #7, #8, #9

S-IC: Understand and evaluate random processes underlying statistical experiments. #1, #2

NCTM Standards

DA: Data: 9-12: understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable

DA: Data: 9-12: understand histograms, parallel box plots, and scatterplots and use them to display data

DA: Statistics: 9-12: identify trends in bivariate data and find functions that model the data

DA: Statistics: 6-8: find, use, and interpret measures of center and spread, including mean and interquartile range

DA: Inferences: 6-8: make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit

Argumentation and Proof

- Logical implication; hypotheses and conclusions; inductive and deductive reasoning. Emphasis should be placed on constructing and recognizing valid mathematical arguments; mathematical proofs should not be considered primarily as formal exercises.

CA Common Core State Standards

Standards for Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.

NCTM Standards

GM: Synthetic: 9-12: establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others

GM: Synthetic: 6-8: create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship

Part 2

Desirable areas of focus for all entering college students (in addition to those in Part 1).

Discrete Mathematics

- Topics such as set theory, graph theory, coding theory, voting systems, game theory, and decision theory.

CA Common Core State Standards

NCTM Standards

GM: Modeling: 9-12: use vertex-edge graphs to model and solve problems

Sequences and Series

- Geometric and arithmetic sequences and series; the Fibonacci sequence; recursion relations.

CA Common Core State Standards

A-SSE: Write expressions in equivalent forms to solve problems. #4

F-IF. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$

for $n \geq 1$. #3F-BF: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. #2

NCTM Standards

Geometry

- *Geometry:* Right triangle trigonometry; transformational geometry including dilations; tessellations; solid geometry; three-dimensional coordinate geometry, including lines and planes.

CA Common Core State Standards

G-SRT: Understand similarity in terms of similarity transformations. #1, #2, #3

G-SRT: Prove theorems involving similarity. #4, #5

G-SRT: Define trigonometric ratios and solve problems involving right triangles. #6, #7, #8

G-GPE: Translate between the geometric description and the equation for a conic section. #1

G-GMD: Explain volume formulas and use them to solve problems. #3

G-GMD: Visualize relationships between two-dimensional and three-dimensional objects. #4

NCTM Standards

GM: Synthetic: 9-12: Use trigonometric relationships to determine lengths and angle measures

GM: Transformations: 9-12: understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, function notation,

GM: Transformations: 9-12: use various representations to help understand the effects of simple transformations and their compositions

Number Theory

- Prime numbers; prime factorization; rational and irrational numbers; triangular numbers; Pascal's triangle; Pythagorean triples.

CA Common Core State Standards

8.NS Know that there are numbers that are not rational, and approximate them by rational numbers. #1, #2

N-RN: Use properties of rational and irrational numbers. #3

A-APR Use Polynomial Identities to Solve Problems. #4

NCTM Standards

NO: Understand numbers: 9-12: compare and contrast the properties of numbers and number systems, including the rational and real numbers-

NO: Understand numbers: 9-12: use number-theory arguments to justify relationships involving whole numbers

NO: Understand numbers: 6-8: use factors, multiples, prime factorization, and relatively prime numbers to solve problems

Part 3

Essential areas of focus for students in quantitative majors (in addition to those in Parts 1 and 2)

Variables, Equations, and Algebraic Expressions

- Solutions to systems of equations, and their geometrical interpretation; solutions to quadratic equations, both algebraic and graphical; complex numbers and their arithmetic; the correspondence between roots and factors of polynomials; rational expressions; the binomial theorem.

CA Common Core State Standards

N-CN: Perform arithmetic operations with complex numbers. #1, #2, #3

N-CN Represent complex numbers and their operations on the complex plane #4, #5

N-CN: Use complex numbers in polynomial identities and equations. #7 #8, #9

A-SSE Interpret the structure of expressions. #2

A-APR: Understand the relationship between the zeros and factors of polynomials. #2, #3

A-APR: Use polynomial identities to solve problems. #4, #5

A-APR: Rewrite rational expressions. #6, #7

A-CED: Create equations that describe numbers or relationships. #2, #3, #4

A-REI: Solve systems of equations. #5, #6

NCTM Standards

NO: Understand numbers: 9-12: compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions

AL: Symbols: 9-12: write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases

Functions

- Rational functions; logarithmic functions, their graphs, and applications; trigonometric functions of real variables, their graphs, properties including periodicity, and applications to right triangle trigonometry; basic trigonometric identities; operations on functions, including addition, subtraction, multiplication, reciprocals, division, composition, and iteration; inverse functions and their graphs; domain and range.

CA Common Core State Standards

F-IF Analyze functions using different representations. #7c,d,e

F-BF Build a function that models a relationship between two quantities #1b,c

F-BF Build new functions form existing functions. #4, #5

F-LE Construct and Compare linear, quadratic and exponential models and solve problems. #2, #3, #4

F-TF: Extend the domain of trigonometric functions using the unit circle. #1, #2, #3, #4

F-TF: Model periodic phenomena with trigonometric functions. #5, #6, #7

F-TF: Prove and apply trigonometric identities. #8, #9

NCTM Standards

AL: Patterns: 9-12: understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions

AL: Patterns: 9-12: understand and compare the properties of classes of functions, including rational, logarithmic, and periodic functions

Geometric Concepts

- Two- and three-dimensional coordinate geometry; locus problems; polar coordinates; vectors; parametric representations of curves.

CA Common Core State Standards

N-VM Represent and model with vector quantities. #1, #2, #3

N-VM Perform operations on vectors. #4, #5

G-GPE Translate between the geometric description and the equation for a conic section. #2, #3

G-GPE Use coordinates to prove simple geometric theorems algebraically. #6, #7

G-GMD Explain volume formulas and use them to solve problems. #1, #2

G-MG: Apply geometric concepts in modeling situations. #1, #2, #3

NCTM Standards

AL: Symbols: 9-12: use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;

GM: Analytic: 9-12: use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations

Argumentation and Proof

- Mathematical implication; mathematical induction and formal proof. Attention should be paid to the distinction between plausible or informal reasoning and complete or rigorous demonstrations.

CA Common Core State Standards

Standards for Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.

NCTM Standards

Part 4

Desirable areas of focus for students in quantitative majors (in addition to those in Part 3)

Vectors and Matrices

- Vectors in the plane; vectors in space; dot and cross product; matrix operations and applications.

CA Common Core State Standards

N-VM: Perform operations on matrices and use matrices in applications. #6, #7, #8, #9, #10, #11, #12

A-REI Solve Systems of equations. #7, #8, #9

NCTM Standards

NO: Understand operations: 9-12: develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices

NO: Compute and estimate: 9-12: develop fluency in operations with vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more complicated cases.

Probability and Statistics

- *Probability and Statistics:* Distributions as models; discrete distributions, such as the Binomial Distribution; continuous distributions, such as the Normal Distribution; fitting data with curves; correlation, regression; sampling, graphical displays of data.

CA Common Core State Standards

S-ID: Summarize, represent, and interpret data on two categorical and quantitative variables. #4, #5, #6

S-IC: Understand and evaluate random processes underlying statistical experiments. #1, #2

S-IC: Make inferences and justify conclusions from sample surveys, experiments, and observational studies #3, #4, #5, #6

NCTM Standards

DA: Data: 9-12: know the characteristics of well-designed studies, including the role of randomization in surveys and experiments

DA: Statistics: 9-12: for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics

DA: Statistics: 9-12: for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools

Conic Sections

- Representations as plane sections of a cone; focus-directrix properties; reflective roperties.

CA Common Core State Standards

G-GPE: Translate between the geometric description and the equation for a conic section. #2, #3

NCTM Standards

Non-Euclidean Geometry

- History of the attempts to prove Euclid's parallel postulate; equivalent forms of the parallel postulate; models in a circle or sphere; seven-point geometry.

CA Common Core State Standards

NCTM Standards

Calculus

- *Calculus:* A high school calculus course should have the same depth, rigor and content as university calculus courses designed for physical sciences and engineering majors. Prior to taking the course, students should have successfully completed four years of secondary school mathematics. Students completing the course should take one of the College Board's Advanced Placement Calculus examinations.

CA Common Core State Standards

NCTM Standards